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PRODUCTION OF TRANSVERSE ENERGY FROM MINIJETS IN NEXT-TO-LEADING ORDER PERTURBATIVE QCD

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Abstract

We compute in next-to-leading order (NLO) perturbative QCD the transverse energy carried into the central rapidity unit of hadron or nuclear collisions by the partons freed in the few-GeV subcollisions. The formulation is based on a rapidity window and a measurement function of a new type. The behaviour of the NLO results as a function of the minimum transverse momentum and as a function of the scale choice is studied. The NLO results are found to be stable relative to the leading-order ones even in the few-GeV domain.

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Below the transverse momenta of observable jets in hadronic collisions, $p_T \lesssim 5$ GeV, but still within the applicability domain of perturbative QCD, $\Lambda_{\rm QCD} \ll p_T$, there is a region of semi-hard parton production, $p_T \sim 1..2$ GeV. In ultrarelativistic heavy ion collisions this region is especially important in the formation of Quark Gluon Plasma at the future colliders BNL-RHIC and CERN-LHC/ALICE: at high collision energies, the few-GeV QCD-quanta, minijets, dominate the initial transverse energy production [1, 2, 3] at central rapidities during the first fractions of fm/c. The initial energy densities for further evolution of the system can thus be estimated based on perturbative QCD (pQCD)[4].

As work pdV is done during the expansion of the system, the initial transverse energy is not, however, directly measurable in AA collisions: the final state E_T in the central rapidity unit has been estimated to be only 1/6 (1/3) of the initially produced E_T at the LHC (RHIC) [4]. The final E_T depends practically linearly on the initial one, so for making reliable estimates of the measurable E_T the initial E_T needs to be computed as accurately as possible.

The average initial transverse energy produced perturbatively in the central rapidity unit ΔY of an AA collision at an impact parameter $\mathbf{b}$ can be computed as [3]

$$\overline{E}_T^{AA}(\mathbf{b},\sqrt{s},p_0,\Delta Y) = T_{AA}(\mathbf{b})\sigma\langle E_T\rangle_{p_0,\Delta Y}. \tag{1}$$

The standard nuclear overlap function $T_{AA}(\mathbf{b})$ accounts for the nuclear collision geometry ($T_{AA}(0) \approx A^2/(\pi R_A^2)$ for $A \sim 200$ [3]). The first moment of the semi-inclusive E_T distribution, $\sigma\langle E_T\rangle_{p_0,\Delta Y}$, is the pQCD quantity we formulate and compute in NLO below. The scale p_0 is the smallest transverse momentum scale in the computation. It also governs the formation time of the system through $\tau_0 \sim 1/p_0$. To compute the actual values of initial E_T, p_0 has to be determined dynamically by introducing additional (nonperturbative) phenomenology, see e.g. [4]. We emphasize that in this work we do not discuss this but aim to show that the NLO pQCD formulation is field-theoretically well defined and perform a rigorous pQCD computation of $\sigma\langle E_T\rangle_{p_0,\Delta Y}$. Therefore, at this level p_0 is a fixed external parameter with no other physical significance than $p_0 \gg \Lambda_{QCD}$. The dependence of $\sigma\langle E_T\rangle$ on p_0 will be explicitly studied and the rigorous NLO computation presented here then sets the stage for more phenomenological analyses.

We generalize the LO formulation [3] of $\sigma\langle E_T\rangle$ to NLO by introducing a new type of infrared (IR) safe measurement functions [5] which contain both the rapidity acceptance and the definition of perturbative collisions. In getting from the $4-2\varepsilon$ dimensional squared matrix elements to the physical quantities we apply the procedure by S. Ellis, Kunszt and Soper (EKS) [5, 6, 7, 8].

The new results obtained in our study can be summarized as follows: A consistent and well-defined NLO pQCD formulation of $\sigma\langle E_T\rangle_{p_0,\Delta Y}$ exists. Eventhough towards the few-GeV region the NLO results could grow rapidly relative to the LO, a stable behaviour of the NLO results is discovered in the range $1...2\,\mathrm{GeV} \lesssim p_0 \lesssim 10\,\mathrm{GeV}$. This

signals of the applicability of the pQCD in the semihard domain. The NLO computation also brings in a new kinematical region not present in the LO, thus increasing the amount of perturbatively computable E_T in nuclear and hadronic collisions. Previously, as the actual NLO contributions were not known, various *ad hoc* K-factors to the LO formulation [3] have been introduced in the literature. We have now rigorously computed the NLO contributions and analysed the obtained K-factors.

We define the acceptance window to coincide with the central rapidity unit, $\Delta Y = \{(y, \phi) : |y| \leq 0.5,\ 0 \leq \phi \leq 2\pi\}$, ϕ being the azimuthal angle and y the rapidity. In a NLO hard scattering of partons, we may have one, two, three or zero partons in our rapidity acceptance. All the partons are assumed massless, so the transverse energy within ΔY is the sum of the absolute values p_{Ti} of the transverse momenta of those partons whose rapidities are in ΔY:

$$E_T = \epsilon(y_1)p_{T1} + \epsilon(y_2)p_{T2} + \epsilon(y_3)p_{T3}, \tag{2}$$

where the step function $\epsilon(y_i)$ is defined as in [3],

$$\epsilon(y_i) \equiv \begin{cases} 1 & \text{if } y_i \in \Delta Y \\ 0 & \text{otherwise.} \end{cases} \tag{3}$$

In the LO, the transverse momenta are equal in magnitude: $p_{T1} = p_{T2} = p_T$. The perturbative scatterings can in this case be simply defined to be those with large enough transverse momentum, $p_T \geq p_0 \gg \Lambda_{\mathrm{QCD}}$, or $p_{T1} + p_{T2} \geq 2p_0$. This generalizes to NLO as

$$p_{T1} + p_{T2} + p_{T3} \geq 2p_0, \tag{4}$$

where p_0 is the *same* external parameter as in the LO case. We emphasize that p_0 above is a *fixed* parameter which does not depend on ΔY.

The IR safe measurement functions S_2 and S_3 [5] can now be written down. They are designed to answer the following question: what is the amount of E_T in ΔY carried by the partons which are produced in scatterings where at least an amount $2p_0$ of transverse momentum is released?

We require both the definition of E_T *and* the definition of hard (perturbative) scatterings to be included in the measurement functions, so for the $2 \to 2$ scatterings we define

$$S_2(p_1, p_2) = \Theta(p_{T1} + p_{T2} \geq 2p_0)\delta(E_T - [\epsilon(y_1)p_{T1} + \epsilon(y_2)p_{T2}]) \tag{5}$$

and for the $2 \to 3$ scatterings correspondingly

$$S_3(p_1, p_2, p_3) = \Theta(p_{T1} + p_{T2} + p_{T3} \geq 2p_0)\delta(E_T - [\epsilon(y_1)p_{T1} + \epsilon(y_2)p_{T2} + \epsilon(y_3)p_{T3}]). \tag{6}$$

We emphasize that these measurement functions are not mere generalizations of the jet cone R of the conventional jet case [5, 6, 7, 8] to a rectangular rapidity window ΔY

but they also carry information of the scale p_0 which is not present in the computation for observable jets. One should also keep in mind that contrary to the E_T of single inclusive jets, the initial E_T we are considering here, is not a directly measurable quantity, especially not for nuclear collisions at RHIC and the LHC. The question that we ask, however, is a well-defined one, and can be answered by making an IR safe pQCD computation analogous to that of observable jets [5].

Extending the definition of the semi-inclusive E_T-distribution of [3] to NLO gives:

$$\begin{aligned}\frac{d\sigma}{dE_T}\Big|_{p_0,\Delta Y} &= \frac{d\sigma}{dE_T}\Big|_{p_0,\Delta Y}^{2\to 2} + \frac{d\sigma}{dE_T}\Big|_{p_0,\Delta Y}^{2\to 3} \\ &= \frac{1}{2!}\int [d\mathrm{PS}]_2 \frac{d\sigma^{2\to 2}}{[d\mathrm{PS}]_2} S_2 + \frac{1}{3!}\int [d\mathrm{PS}]_3 \frac{d\sigma^{2\to 3}}{[d\mathrm{PS}]_3} S_3 \end{aligned} \tag{7}$$

in which momentum conservation correlates the transverse momenta as $\mathbf{p}_{T1} = -\mathbf{p}_{T2}$ in the $2 \to 2$ kinematics, and $\mathbf{p}_{T1} = -(\mathbf{p}_{T2} + \mathbf{p}_{T3})$ in the $2 \to 3$ kinematics. The $[d\mathrm{PS}]_2$ and $[d\mathrm{PS}]_3$ stand for $dp_{T2}dy_1dy_2$ and $dp_{T2}dp_{T3}d\phi_2 d\phi_3 dy_1 dy_2 dy_3$, respectively.

The divergencies present in the partonic NLO cross sections can be regulated by computing the squared matrix elements in $4-2\varepsilon$ dimensions, resulting in an explicit ε^{-1} and ε^{-2} behaviour of the divergent terms. This was done first by R.K. Ellis and Sexton [10]. Using the $4-2\varepsilon$ dimensional squared matrix elements EKS have formulated the calculation of observable cross sections for jet production [5, 6, 7, 8]. We will apply their approach, based on the subtraction method, here.

As explained in [5], cancellation of the divergencies takes place only if the three-parton measurement function S_3 reduces to the two-parton one S_2 in soft and collinear limits. Our measurement functions in Eqs. (5) and (6) clearly fulfil these criteria. It is also worth mentioning that while the transverse energy is a good quantity to compute, e.g. the number of gluons in ΔY would not make an IR safe measurement function without specifying when two nearly collinear gluons are to be counted as one.

From Eq. (7) one obtains the first moment of the semi-inclusive E_T distribution:

$$\begin{aligned}\sigma\langle E_T\rangle_{p_0,\Delta Y} &\equiv \int_0^{\sqrt{s}} dE_T\, E_T \frac{d\sigma}{dE_T}\Big|_{p_0,\Delta Y} \\ &= \sigma\langle E_T\rangle_{p_0,\Delta Y}^{2\to 2} + \sigma\langle E_T\rangle_{p_0,\Delta Y}^{2\to 3}, \end{aligned} \tag{8}$$

where, after integrating the delta functions away in Eq. (7),

$$\sigma\langle E_T\rangle_{p_0,\Delta Y}^{2\to 2} = \frac{1}{2!}\int [d\mathrm{PS}]_2 \frac{d\sigma^{2\to 2}}{[d\mathrm{PS}]_2} \tilde{S}_2(p_1,p_2) \tag{9}$$

and

$$\sigma\langle E_T\rangle_{p_0,\Delta Y}^{2\to 3} = \frac{1}{3!}\int [d\mathrm{PS}]_3 \frac{d\sigma^{2\to 3}}{[d\mathrm{PS}]_3} \tilde{S}_3(p_1,p_2,p_3). \tag{10}$$

The measurement functions for the first E_T-moment above are denoted by

$$\tilde{S}_2(p_1,p_2) = \Big[\epsilon(y_1)+\epsilon(y_2)\Big] p_{T2}\Theta(p_{T2}\geq p_0) \tag{11}$$

and

$$\tilde{S}_3(p_1,p_2,p_3) = \Big[\epsilon(y_1)p_{T1}+\epsilon(y_2)p_{T2}+\epsilon(y_3)p_{T3}\Big]\Theta(p_{T1}+p_{T2}+p_{T3}\geq 2p_0), \tag{12}$$

where $p_{T1} = |\mathbf{p}_{T2}+\mathbf{p}_{T3}|$. Naturally, also $\tilde{S}_2$ and $\tilde{S}_3$ fulfil the IR criteria, which ensures that $\sigma\langle E_T\rangle_{p_0,\Delta Y}$ is a well-defined IR safe quantity to compute. From the general form of Eqs. (9) and (10) we notice that, by replacing the measurement functions S_2 and S_3 of Ref. [5] by $\tilde{S}_2$ and $\tilde{S}_3$ above, we can follow the formulation of the problem as given in Ref. [5]. As all the relevant formulae with a complete discussion of the cancellation of the several $\sim\varepsilon^{-1}$ and $\sim\varepsilon^{-2}$ singularities can be found in all detail in [5], we now proceed directly to the numerical evaluation.

Our $\tilde{S}_2$ and $\tilde{S}_3$ are azimuthally symmetric, the ϕ_2-integrals giving a trivial factor 2π, so basically only three- and six-dimensional integrals remain to be done in $\sigma\langle E_T\rangle^{2\to 2}_{p_0,\Delta Y}$ and $\sigma\langle E_T\rangle^{2\to 3}_{p_0,\Delta Y}$, correspondingly. For $2\to 2$ kinematics, the integration limits imposed by the acceptance cuts in $\tilde{S}_2$ can be solved analytically, allowing for a quick and accurate computation using a NAG library [11] subroutine based on an adaptive subdivision strategy.

Contrary to the $2\to 2$ case, the kinematical cuts for the three particle phase space are very complicated due to the additional cuts introduced in the subtraction terms. To keep the counterparts of the subtraction terms as they are given in [5] (and which have $2\to 2$ kinematics), the kinematical cuts for the subtraction terms in the $2\to 3$ integrals must be strictly imposed through the measurement functions $\tilde{S}_3$. These integrals are evaluated using a Monte Carlo subroutine of NAG.

Since the subtraction procedure adopted here is exactly that of Refs. [5, 6, 7] for production of inclusive high-p_T jets, we are also able to make use of certain subroutines in the JET program [12] of EKS. In particular, we have used the subroutines of JET for the functions in terms of which the squared matrix elements were expressed in [5]. In addition, we have adopted EKS's book-keeping method of the parton flavours, and explicitly made use of their permutation tables for different subprocesses.

For the NLO $\overline{\rm MS}$ parton distributions, we use the GRV94-HO [13] as implemented in PDFLIB [14]. The renormalization scale and factorization scale are chosen equal, $\mu_R=\mu_F=\mu$. Our choice for the $2\to 3$ terms is a generalization of the $2\to 2$ case $\mu = N_\mu\times p_T$,

$$\mu = N_\mu \times (p_{T1}+p_{T2}+p_{T3})/2 \tag{13}$$

where N_μ is a number of the order of unity. The scale μ is therefore IR safe in the same sense as the measurement functions are, as required by the cancellation of the divergencies.

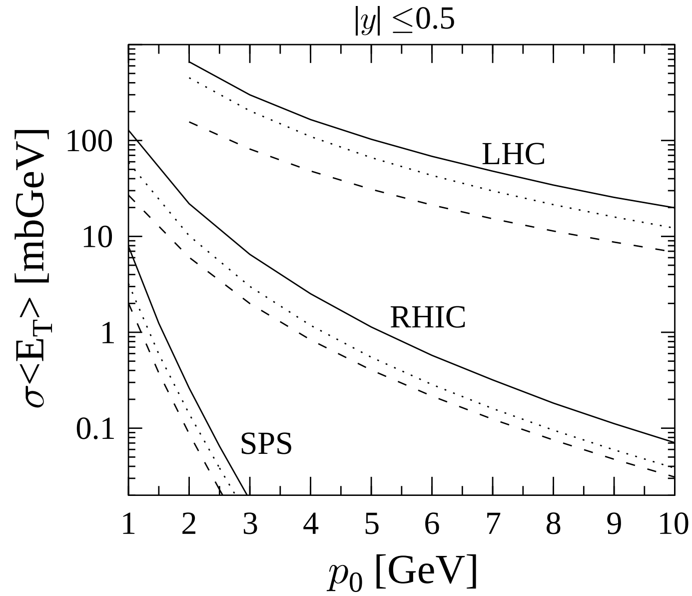

Figure 1: The first E_T moment $\sigma\langle E_T\rangle_{p_0,\Delta Y}$ in the central rapidity unit $|y| \leq 0.5$ of the E_T distribution (8). The results are shown for pp collisions at $\sqrt{s} = 5.5$ TeV (LHC), 200 GeV (RHIC) and 20 GeV (SPS) as functions of the parameter p_0. The solid curves represent the full NLO calculation, the dotted ones show the LO results computed with GRV94-LO parton distributions and one-loop α_s, and the dashed curves stand for the LO computation with GRV94-HO parton distributions and two-loop α_s. All the scales are chosen (Eq. 13) as $\mu = (p_{T1} + p_{T2} + p_{T3})/2$.

In Fig. 1 we plot the first E_T moment $\sigma\langle E_T\rangle_{p_0,\Delta Y}$ in pp collisions (i.e. no nuclear shadowing is included) in the central rapidity unit $|y| \leq 0.5$, as a function of the minimum transverse momentum p_0 for $\sqrt{s} = 5.5$ TeV, 200 GeV and 20 GeV. Based on Ref. [4], for AA collisions ($A \sim 200$) we expect the relevant range of p_0 to be $p_0 \sim 2(1)$ GeV for the LHC (RHIC), so we show $p_0 = 1...10$ GeV in the figure to see whether the NLO results become very unstable relative to LO at the few-GeV transverse momenta. The difference between the two LO results at $\sqrt{s} = 200$ GeV is mainly due to the difference between the one- and two-loop $\alpha_s(\mu)$ used. At $\sqrt{s} = 5500$ GeV the differencies of the NLO and LO parton distributions also become important. The K-factors, defined as K =(full NLO)/LO, with LO in the denominator being either the dashed or dotted curve, can now be directly read off from the figure: At RHIC, as p_0 is decreased from 10 to 1 GeV, the K-factors range from 2.3 to 4.8 and from 1.9 to 2.2, the latter being the one evaluated against the truly LO result, i.e. the dotted line. At the LHC the corresponding values are from 3.0 to 4.2 and from 1.6 to 1.5 when p_0 is varied from 10 GeV down to 2 GeV.

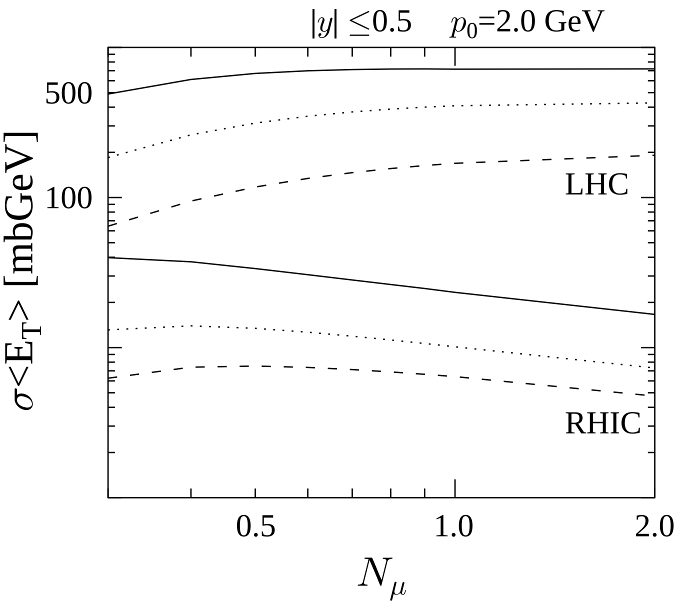

Figure 2: Scale dependence of $\sigma\langle E_T\rangle_{p0,\Delta Y}$ as a function of $N_\mu = 2\mu/(p_{T1}+p_{T2}+p_{T3})$ for the LHC (upper set of three curves) and RHIC (the three lowest curves) energies at fixed $p_0 = 2$ GeV. Labelling of the curves is the same as in Fig. 1.

In light of the inclusive one-jet production, [7], the K-factors are expected to depend on the scale choice μ. In the best case scenario from the viewpoint of applicability of pQCD at $p_0 \sim$ a few GeV, a region in $N_\mu \sim \mathcal{O}(1)$ would exist where the NLO results would be stable against variations of the scale choice, i.e. independent of N_μ in Eq. (13). This is studied in Fig. 2, where the full NLO results and the two LO calculations are shown as a function of N_μ for a fixed value of $p_0 = 2$ GeV. The values of the curves at $N_\mu = 1$ are the same as in the previous figure at $p_0 = 2$ GeV. At RHIC energies, the scale dependence of the NLO-results is very similar to the LO case. For the LHC, the NLO results do *not* depend much on the scale choice. A similar behaviour is, however, found also in the LO, so the stable behaviour cannot be attributed to a better convergence of the perturbation series obtained by including the NLO terms. The stable behaviour is due to the decrease of $\alpha_s(\mu)$ and the increase of $xg(x,\mu^2)$ with larger values of N_μ. In the p_0-region considered here, these effects virtually cancel. The full NLO results seem therefore to be as (in)sensitive to the choice of the scale as the LO results are both at the LHC and RHIC energies at $p_0 \sim$ a few GeV.

The following remarks are in order: First, consider the K-factor calculated with respect to the dashed curve in Fig. 1. We note that this ratio is quite stable, but presents a small increase towards smaller values of p_0. This signals that we are working near the borderline of perturbative QCD. On the other hand the K-factor calculated

against the truly LO result, shown by the dotted curve in Fig. 1, does not increase, which implies that the NLO results are stable all the way down to $p_0 = 1...2$ GeV. Second, the actual magnitude of the K-factors partly follows from the fact that below the value $E_T = p_0$ there is a completely new contribution to the minijet E_T introduced by the NLO terms: the E_T-distribution at $E_T < p_0$ is empty in LO but in NLO this is not the case anymore. Therefore, more attention shoud be paid to the overall behaviour than to the absolute magnitude of the K-factors. The presence of the new kinematical region in NLO allows us to conjecture that since this region would be present also in the order next to NLO, the NNLO terms will be suppressed. Third, based on the jet cone R dependence of the inclusive jet production [7] one might expect that the K-factors discussed above depend on the choice for ΔY. However, as the LO contribution to $\sigma\langle E_T\rangle_{p_0,\Delta Y}$ is almost linearly proportional to ΔY, the ΔY dependence of the K-factors should become much weaker than in the inclusive one-jet case.

An attempt to compute $\sigma\langle E_T\rangle_{p_0,\Delta Y}$ in NLO with the semi-hard region included has also been recently presented in Ref. [9]. The formulation [9] is, however, a more direct generalization of the inclusive jet production [5, 6, 7, 8]: ΔY is introduced and E_T within ΔY is defined as we have done above but a cut-off E_0 in E_T is introduced. Consequently, at $E_T < E_0$, the E_T distribution is empty both in LO and especially in NLO. The formulation [9] thus misses the new, calculable, perturbative contribution in this region. In addition, a cut-off imposed in E_T causes a ΔY dependence of p_0 not present in our formulation. We believe that to maximally account for the E_T production from minijets, and thus to correctly generalize the formulation of Ref. [3], the starting point must be to first fix the minimum overall transverse momentum $2p_0$ once and for all, regardless whether the partons fall into ΔY or not. As discussed above, this results in more than just an extension of the conventional jet calculation.

We have in this letter extended the LO formalism of Ref. [3] to NLO. Comparison against LO results, taken together with the fact that a new kinematical region is contained in NLO, shows that the NLO computation is a meaningful one, and should be taken into account when evaluating initial transverse energy in nuclear collisions.

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